



PRINCIPIO
MATHEMATICS

Fractions Practice Book

Meaning / Operations / Transfer / Cumulative review

Verified mathematics, purposeful models, fading support, and spaced retrieval

14 learning units / 122 activities

HOW THIS BOOK WORKS

See a model or worked example when the concept needs it.

Practice with support that fades instead of staying permanent.

Retrieve earlier ideas later, then use them in mixed contexts.

FREE PRACTICE BOOK

Learn without gaps.

Verified Practice Edition

Practice with progression

Attempt learner-response activities before using the verified solutions appendix.

Retrieval activities intentionally point backward; do not treat them as new instruction.

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UNIT 01

What Is a Fraction?

WORKED EXAMPLE

Study the bar. What does $\frac{3}{4}$ mean when the whole is divided into equal parts?

Study the verified reasoning before independent work.
Use the model. Use the vocabulary cue.

MATHEMATICAL MODEL

$\frac{3}{4}$



VERIFIED REASONING

- 1 The whole is divided into 4 equal parts.
- 2 Three of those equal parts are selected, so the fraction is $\frac{3}{4}$.



CHECK YOUR UNDERSTANDING

Using the bar, explain what the 3 and the 4 each tell us in $\frac{3}{4}$.

Explain the mathematical relationship, not only the result.
Use the model.

MATHEMATICAL MODEL

$\frac{3}{4}$



EXPLAIN WITH MATHEMATICAL EVIDENCE

GUIDED PRACTICE

A whole is divided into 5 equal parts and 2 are selected. Write the fraction.

Use the visible support, then complete the mathematics.
Use the vocabulary cue. Start with the first step shown.

PLAN / WORK / CHECK



INDEPENDENT PRACTICE

A whole has 7 equal parts and 4 are selected. Write the fraction without a hint.

Solve without a worked answer on this page.

WORK INDEPENDENTLY

APPLY & CONNECT

The point on the number line is $\frac{3}{4}$. Explain how that location expresses the same equal-parts meaning as a fraction bar.

Connect the context or model to a mathematical statement.
Use the model.

MATHEMATICAL MODEL



MODEL OR CONTEXT / MATHEMATICS / INTERPRETATION



UNIT 02

Numerator and Denominator

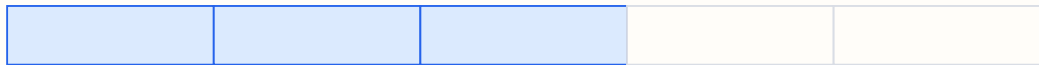
WORKED EXAMPLE

Study $\frac{3}{5}$ and the bar. Identify the numerator and denominator and state each role.

Study the verified reasoning before independent work.
Use the model. Use the vocabulary cue.

MATHEMATICAL MODEL

$\frac{3}{5}$



VERIFIED REASONING

- 1 In $\frac{3}{5}$, 3 is the numerator and 5 is the denominator.
- 2 The denominator sets the equal-part size; the numerator counts selected parts.



CHECK YOUR UNDERSTANDING

Why does the denominator 5 in $\frac{3}{5}$ not mean that five pieces are selected?

Explain the mathematical relationship, not only the result.
Use the model.

MATHEMATICAL MODEL

$\frac{3}{5}$



EXPLAIN WITH MATHEMATICAL EVIDENCE

SPOT THE ERROR

CLAIM TO CHECK - FIND AND REPAIR THE ERROR

A student says: 'In $\frac{3}{5}$, both 3 and 5 count selected pieces.' What is wrong with that explanation?

Inspect the claim, identify the error, and correct it.

DIAGNOSE / CORRECT / EXPLAIN



GUIDED PRACTICE

For $\frac{4}{7}$, label numerator and denominator, then finish: the denominator tells _____; the numerator tells _____.

Use the visible support, then complete the mathematics.
Use the vocabulary cue. Complete the missing steps.

PLAN / WORK / CHECK

INDEPENDENT PRACTICE

Explain the roles of 5 and 8 in $\frac{5}{8}$ without using a hint.

Solve without a worked answer on this page.

WORK INDEPENDENTLY



UNIT 03

Representing Fractions

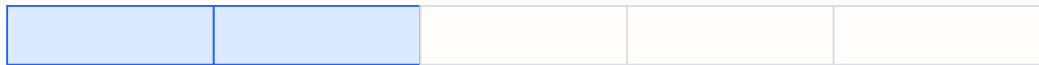
WORKED EXAMPLE

Study the bar and connect its five equal parts and two shaded parts to the symbol $\frac{2}{5}$.

Study the verified reasoning before independent work.
Use the model. Use the vocabulary cue.

MATHEMATICAL MODEL

$\frac{2}{5}$



VERIFIED REASONING

- 1 The bar has five equal parts and two are selected.
- 2 That model represents $\frac{2}{5}$.



CHECK YOUR UNDERSTANDING

Why does this bar represent $\frac{2}{5}$ rather than $\frac{5}{2}$?

Explain the mathematical relationship, not only the result.
Use the model.

MATHEMATICAL MODEL

$\frac{2}{5}$



EXPLAIN WITH MATHEMATICAL EVIDENCE

GUIDED PRACTICE

A model is divided into 8 equal parts and 3 are selected. Complete the symbol: $\frac{\quad}{\quad}$.

Use the visible support, then complete the mathematics.
Use the vocabulary cue. Complete the missing steps.

PLAN / WORK / CHECK



INDEPENDENT PRACTICE

Write the fraction represented by a whole divided into 5 equal parts with 2 selected.

Solve without a worked answer on this page.

WORK INDEPENDENTLY

APPLY & CONNECT

The number line marks $\frac{2}{5}$. Explain why that point and a bar with two of five equal parts shaded represent the same number.

Connect the context or model to a mathematical statement.
Use the model.

MATHEMATICAL MODEL



MODEL OR CONTEXT / MATHEMATICS / INTERPRETATION



QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back: in $\frac{3}{7}$, what does the denominator 7 mean and what does the numerator 3 mean?

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



UNIT 04

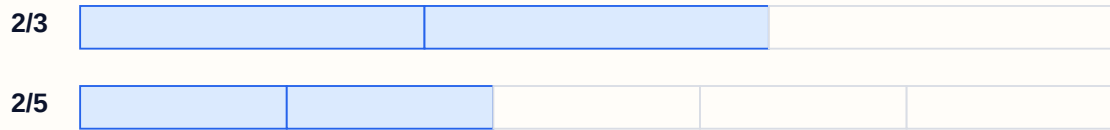
Comparing Simple Fractions

WORKED EXAMPLE

Study the two equal-length bars. Which is greater, $\frac{2}{3}$ or $\frac{2}{5}$, and what does the picture show?

Study the verified reasoning before independent work.
Use the model. Use the vocabulary cue.

MATHEMATICAL MODEL



VERIFIED REASONING

- 1 Both fractions use the same-sized whole.
- 2 Two thirds covers more of that whole than two fifths, so $\frac{2}{3} > \frac{2}{5}$.



CHECK YOUR UNDERSTANDING

Use the number line to explain why $\frac{2}{3}$ is greater than $\frac{2}{5}$.

Explain the mathematical relationship, not only the result.
Use the model.

MATHEMATICAL MODEL



EXPLAIN WITH MATHEMATICAL EVIDENCE

GUIDED PRACTICE

Compare $\frac{3}{4}$ and $\frac{2}{3}$. Use a common scale or cross-products, then write $>$, $<$, or $=$.

Use the visible support, then complete the mathematics.
Hint: use the suggested operation. Start with the first step shown.

PLAN / WORK / CHECK



INDEPENDENT PRACTICE

Compare $\frac{5}{8}$ and $\frac{3}{4}$ without a hint. Write the correct relation and justify it exactly.

Solve without a worked answer on this page.

WORK INDEPENDENTLY

SPOT THE ERROR

CLAIM TO CHECK - FIND AND REPAIR THE ERROR

A student says $\frac{2}{3} < \frac{2}{5}$ because 5 is the larger denominator. Diagnose the error.

Inspect the claim, identify the error, and correct it.

DIAGNOSE / CORRECT / EXPLAIN

CHOOSE A METHOD

To compare $\frac{3}{4}$ and $\frac{2}{3}$ reliably, choose: A) compare denominators only; B) compare numerators only; C) use a common exact scale or cross-products. Explain why.

Choose a method and justify why it fits the relationship.

YOUR CHOICE / WHY THIS METHOD?

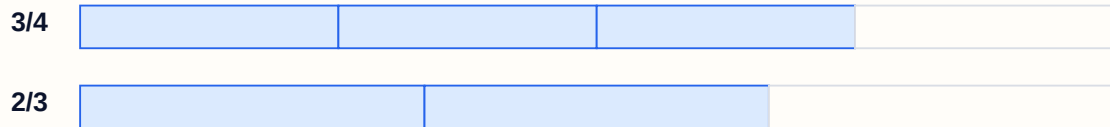


APPLY & CONNECT

The bars show $\frac{3}{4}$ and $\frac{2}{3}$ of equal wholes. Which fraction is larger? Connect the visual model to an exact symbolic comparison.

Connect the context or model to a mathematical statement.
Use the model.

MATHEMATICAL MODEL



MODEL OR CONTEXT / MATHEMATICS / INTERPRETATION

QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back: in $\frac{5}{9}$, identify numerator and denominator and state their different roles.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



UNIT 05

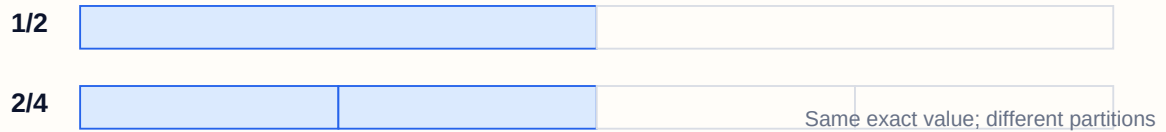
Equivalent Fractions

WORKED EXAMPLE

Study the two fraction bars. They look partitioned differently. What relationship do they show?

Study the verified reasoning before independent work.
Use the model. Use the vocabulary cue.

MATHEMATICAL MODEL



VERIFIED REASONING

- 1 Both bars cover exactly one half of the same-sized whole.
- 2 Different numbers of equal parts can represent the same value.
- 3 Therefore $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.



CHECK YOUR UNDERSTANDING

Do $\frac{1}{2}$ and $\frac{2}{4}$ represent the same amount? Explain using the bars.

Explain the mathematical relationship, not only the result.
Use the model.

MATHEMATICAL MODEL



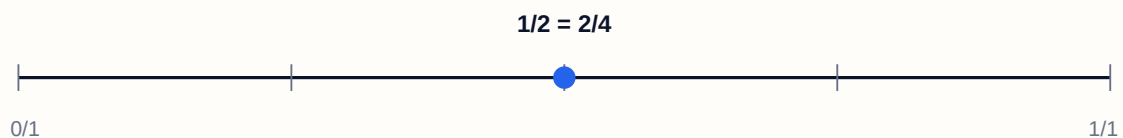
EXPLAIN WITH MATHEMATICAL EVIDENCE

CHECK YOUR UNDERSTANDING

On the number line, should $\frac{1}{2}$ and $\frac{2}{4}$ appear at different points or the same point? Why?

Explain the mathematical relationship, not only the result.
Use the model.

MATHEMATICAL MODEL



EXPLAIN WITH MATHEMATICAL EVIDENCE



GUIDED PRACTICE

Complete the equivalent fraction: $\frac{3}{5} = \frac{\quad}{10}$. What factor changes 5 into 10?

Use the visible support, then complete the mathematics.
Hint: use the suggested operation.

PLAN / WORK / CHECK

INDEPENDENT PRACTICE

Write $\frac{4}{7}$ as an equivalent fraction with denominator 21.

Solve without a worked answer on this page.

WORK INDEPENDENTLY

SPOT THE ERROR

CLAIM TO CHECK - FIND AND REPAIR THE ERROR

A student says $\frac{3}{4} = \frac{4}{5}$ because adding 1 to both numerator and denominator preserves equivalence. Diagnose the error and correct the claim.

Inspect the claim, identify the error, and correct it.

DIAGNOSE / CORRECT / EXPLAIN



CHOOSE A METHOD

Which method keeps $\frac{3}{4}$ equivalent? A) add 1 to numerator and denominator; B) multiply both by 2; C) multiply only the numerator by 2.

Choose a method and justify why it fits the relationship.

YOUR CHOICE / WHY THIS METHOD?

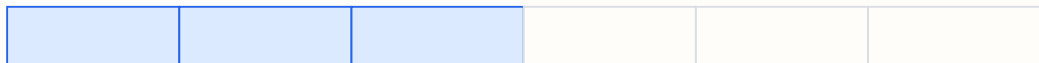
APPLY & CONNECT

The model shows $\frac{3}{6}$ of a whole. Which simpler fraction names the same amount?

Connect the context or model to a mathematical statement.
Use the model.

MATHEMATICAL MODEL

$\frac{3}{6}$



MODEL OR CONTEXT / MATHEMATICS / INTERPRETATION



QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, represent $\frac{3}{5}$ as equal parts of one whole and state how many of those parts are selected.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



UNIT 06

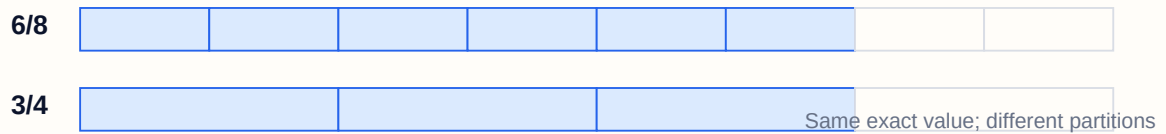
Simplifying Fractions

WORKED EXAMPLE

Study how $\frac{6}{8}$ becomes $\frac{3}{4}$ without changing the shaded amount.

Study the verified reasoning before independent work.
Use the model. Start with the first step shown.

MATHEMATICAL MODEL



VERIFIED REASONING

- 1 The greatest common divisor of 6 and 8 is 2.
- 2 Divide numerator and denominator by the same nonzero factor: $(6 \div 2) / (8 \div 2) = \frac{3}{4}$.
- 3 The visual model confirms that the value did not change.

GUIDED PRACTICE

Simplify $\frac{12}{18}$. First identify a common factor, then divide numerator and denominator by it.

Use the visible support, then complete the mathematics.
Start with the first step shown. Complete the missing steps.

PLAN / WORK / CHECK



GUIDED PRACTICE

Simplify $15/20$ to lowest terms. Use the greatest common factor if you can find it.

Use the visible support, then complete the mathematics.
Hint: use the suggested operation.

PLAN / WORK / CHECK

INDEPENDENT PRACTICE

Simplify $21/28$ to lowest terms.

Solve without a worked answer on this page.
Use the checklist.

WORK INDEPENDENTLY

INDEPENDENT PRACTICE

Simplify $36/48$ to lowest terms.

Solve without a worked answer on this page.

WORK INDEPENDENTLY



SPOT THE ERROR

CLAIM TO CHECK - FIND AND REPAIR THE ERROR

A student says $\frac{6}{8}$ simplifies to $\frac{3}{8}$ because only the numerator needs to be divided by 2. Diagnose the error and give the correct lowest terms.

Inspect the claim, identify the error, and correct it.

DIAGNOSE / CORRECT / EXPLAIN

CHOOSE A METHOD

To simplify $\frac{18}{24}$ to lowest terms in one step, which common factor should you divide by? A) 2, B) 3, C) 6.

Choose a method and justify why it fits the relationship.

YOUR CHOICE / WHY THIS METHOD?



APPLY & CONNECT

The bar model represents $\frac{12}{16}$ of a whole. Write the same amount as a fraction in lowest terms.

Connect the context or model to a mathematical statement.
Use the model.

MATHEMATICAL MODEL



MODEL OR CONTEXT / MATHEMATICS / INTERPRETATION

QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, compare $\frac{3}{5}$ and $\frac{2}{3}$ on a common scale and state the exact relation.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



UNIT 07

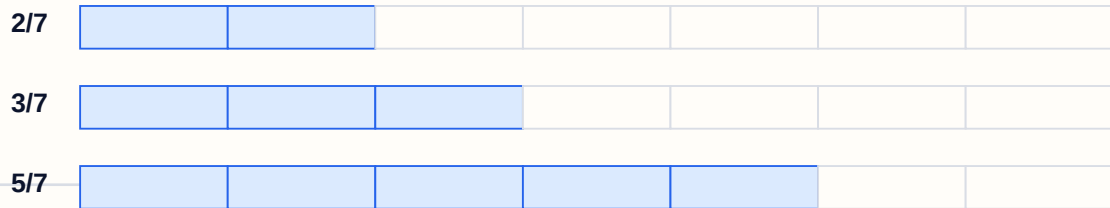
Adding Fractions with Equal Denominators

WORKED EXAMPLE

Study the seventh-bars for $\frac{2}{7}$, $\frac{3}{7}$, and $\frac{5}{7}$. What stays unchanged when the two fractions are added?

Study the verified reasoning before independent work.
Use the model. Use the vocabulary cue.

MATHEMATICAL MODEL



VERIFIED REASONING

- 1 Both fractions use sevenths, so the pieces already have the same size.
- 2 Combine the selected pieces: $2 + 3 = 5$ while the denominator remains 7.
- 3 Therefore $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$.

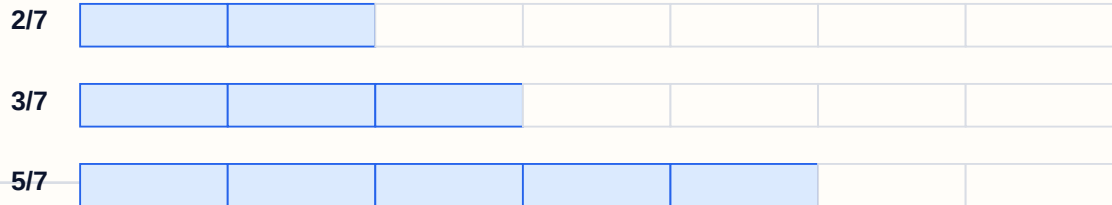


CHECK YOUR UNDERSTANDING

Why does the denominator remain 7 in $2/7 + 3/7$? Explain using equal-sized parts.

Explain the mathematical relationship, not only the result.
Use the model.

MATHEMATICAL MODEL



EXPLAIN WITH MATHEMATICAL EVIDENCE

WORKED EXAMPLE

Study the worked reasoning for $1/8 + 4/8$. Why can the numerators be combined immediately?

Study the verified reasoning before independent work.
Start with the first step shown. Use the vocabulary cue.

VERIFIED REASONING

- 1 Model the procedure: keep denominator 8 and add numerators $1 + 4$.
- 2 The result is $5/8$.



GUIDED PRACTICE

Add $\frac{3}{9} + \frac{2}{9}$. Use the common denominator and complete the numerator step.

Use the visible support, then complete the mathematics.

Hint: use the suggested operation. Start with the first step shown.

PLAN / WORK / CHECK

GUIDED PRACTICE

Add $\frac{1}{6} + \frac{4}{6}$. Decide what changes and what remains fixed.

Use the visible support, then complete the mathematics.

Hint: use the suggested operation.

PLAN / WORK / CHECK

SPOT THE ERROR

CLAIM TO CHECK - FIND AND REPAIR THE ERROR

A student writes $\frac{2}{7} + \frac{3}{7} = \frac{5}{14}$. Identify the error and give the correct sum.

Inspect the claim, identify the error, and correct it.

DIAGNOSE / CORRECT / EXPLAIN



INDEPENDENT PRACTICE

Add $5/12 + 4/12$ and give the final answer in lowest terms.

Solve without a worked answer on this page.

WORK INDEPENDENTLY

APPLY & CONNECT

The model shows the result of $2/8 + 3/8$. Connect the symbolic sum to the shaded amount and state the result.

Connect the context or model to a mathematical statement.
Use the model.

MATHEMATICAL MODEL

$5/8$



MODEL OR CONTEXT / MATHEMATICS / INTERPRETATION



QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, write $\frac{3}{4}$ as an equivalent fraction with denominator 12.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



UNIT 08

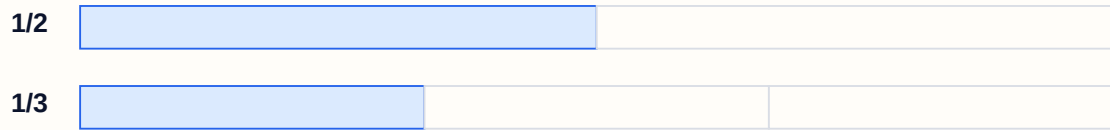
Adding Fractions with Unlike Denominators

WORKED EXAMPLE

Study the $\frac{1}{2}$ and $\frac{1}{3}$ bars. Why can their numerators not be combined immediately?

Study the verified reasoning before independent work.
Use the model. Use the vocabulary cue.

MATHEMATICAL MODEL



VERIFIED REASONING

- 1 Halves and thirds are different-sized parts, so their numerators cannot be combined yet.
- 2 Rewrite them on a common scale: $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$.
- 3 Then $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.

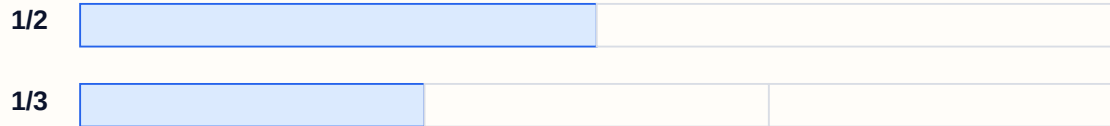


CHECK YOUR UNDERSTANDING

Explain why a common denominator is about equal-sized parts, not merely matching symbols.

Explain the mathematical relationship, not only the result.
Use the model.

MATHEMATICAL MODEL



EXPLAIN WITH MATHEMATICAL EVIDENCE

WORKED EXAMPLE

Study the worked addition $\frac{1}{3} + \frac{1}{4}$. Follow the equivalent-fraction conversion before adding.

Study the verified reasoning before independent work.
Start with the first step shown. Hint: use the suggested operation.

VERIFIED REASONING

- 1 Use denominator 12: $\frac{1}{3} = \frac{4}{12}$ and $\frac{1}{4} = \frac{3}{12}$.
- 2 Then $\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$.



GUIDED PRACTICE

Add $\frac{1}{2} + \frac{1}{6}$. First rewrite the half using sixths.

Use the visible support, then complete the mathematics.
Start with the first step shown. Complete the missing steps.

PLAN / WORK / CHECK

GUIDED PRACTICE

Add $\frac{2}{5} + \frac{1}{10}$. Choose a common denominator and complete the sum.

Use the visible support, then complete the mathematics.
Choose from the suggested methods.

PLAN / WORK / CHECK

SPOT THE ERROR

CLAIM TO CHECK - FIND AND REPAIR THE ERROR

A student says $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$. Diagnose the error and repair the calculation.

Inspect the claim, identify the error, and correct it.

DIAGNOSE / CORRECT / EXPLAIN



CHOOSE A METHOD

For $\frac{1}{3} + \frac{1}{4}$, which proposed common denominator works? A) 7, B) 12, C) 20. Justify the choice.

Choose a method and justify why it fits the relationship.

YOUR CHOICE / WHY THIS METHOD?

INDEPENDENT PRACTICE

Add $\frac{3}{8} + \frac{1}{6}$ and give the exact result in lowest terms.

Solve without a worked answer on this page.

WORK INDEPENDENTLY

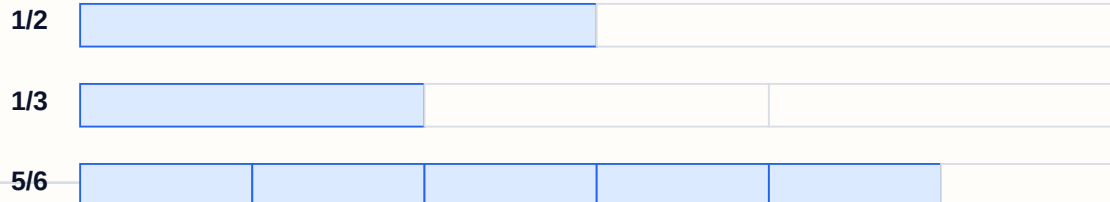


APPLY & CONNECT

Use the bars to connect $\frac{1}{2} + \frac{1}{3}$ with the final amount $\frac{5}{6}$. Explain the common-scale step.

Connect the context or model to a mathematical statement.
Use the model.

MATHEMATICAL MODEL



MODEL OR CONTEXT / MATHEMATICS / INTERPRETATION

QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, simplify $\frac{12}{16}$ to lowest terms.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



UNIT 09

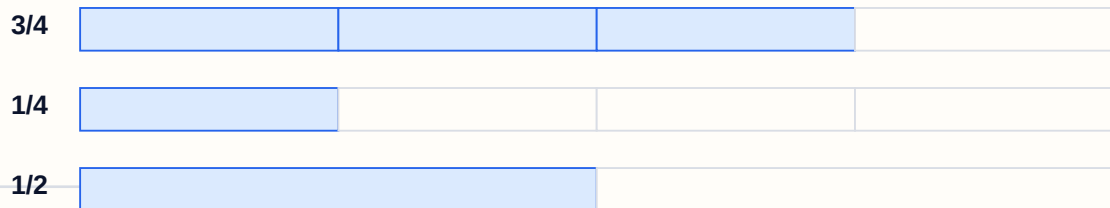
Subtracting Fractions

WORKED EXAMPLE

Study $\frac{3}{4} - \frac{1}{4}$ with the bars. What happens to the selected fourths, and how does the result simplify?

Study the verified reasoning before independent work.
Use the model. Start with the first step shown.

MATHEMATICAL MODEL



VERIFIED REASONING

- 1 Both fractions use fourths, so subtract the selected fourths.
- 2 $\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$.

GUIDED PRACTICE

Subtract $\frac{5}{6} - \frac{1}{3}$. First rewrite the third using sixths.

Use the visible support, then complete the mathematics.
Start with the first step shown. Complete the missing steps.

PLAN / WORK / CHECK



GUIDED PRACTICE

Subtract $7/8 - 1/4$. Choose an equivalent form for $1/4$ before subtracting.

Use the visible support, then complete the mathematics.
Choose from the suggested methods.

PLAN / WORK / CHECK

CHOOSE A METHOD

For $3/4 - 1/6$, which listed common denominator is the best choice? A) 10, B) 12, C) 24. Justify.

Choose a method and justify why it fits the relationship.

YOUR CHOICE / WHY THIS METHOD?

SPOT THE ERROR

CLAIM TO CHECK - FIND AND REPAIR THE ERROR

A student writes $3/4 - 1/2 = 2/2$. Diagnose the error and calculate the exact difference.

Inspect the claim, identify the error, and correct it.

DIAGNOSE / CORRECT / EXPLAIN



INDEPENDENT PRACTICE

Subtract $11/12 - 2/9$ and give the exact result in lowest terms.

Solve without a worked answer on this page.

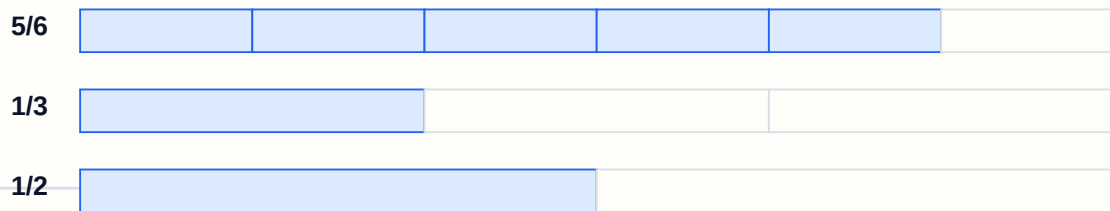
WORK INDEPENDENTLY

APPLY & CONNECT

Use the model to explain why $5/6 - 1/3$ equals $1/2$ after equivalent conversion and simplification.

Connect the context or model to a mathematical statement.
Use the model.

MATHEMATICAL MODEL



MODEL OR CONTEXT / MATHEMATICS / INTERPRETATION



QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, write $\frac{2}{3}$ as an equivalent fraction with denominator 12.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY

QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, add $\frac{3}{10} + \frac{4}{10}$.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



UNIT 10

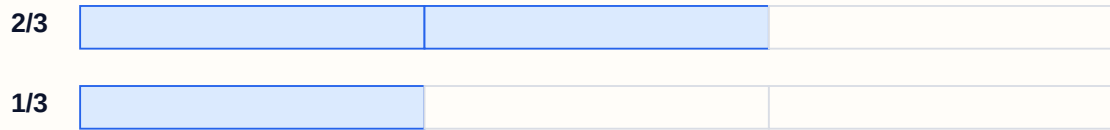
Multiplying Fractions

WORKED EXAMPLE

Study the bars for $\frac{2}{3}$ and $\frac{1}{3}$. Why does taking one half of $\frac{2}{3}$ produce $\frac{1}{3}$?

Study the verified reasoning before independent work.
Use the model. Use the vocabulary cue.

MATHEMATICAL MODEL



VERIFIED REASONING

- 1 Two thirds of the whole is the starting amount.
- 2 Taking one half of that amount leaves one third of the whole.
- 3 Exactly: $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$.

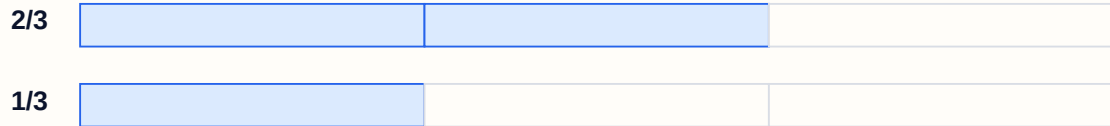


CHECK YOUR UNDERSTANDING

Using the bars, explain why the word 'of' in one half of two thirds can be represented by multiplication.

Explain the mathematical relationship, not only the result.
Use the model.

MATHEMATICAL MODEL



EXPLAIN WITH MATHEMATICAL EVIDENCE

WORKED EXAMPLE

Study the verified procedure for $\frac{2}{3} \times \frac{3}{5}$. Track the unsimplified product before reducing it.

Study the verified reasoning before independent work.
Start with the first step shown. Hint: use the suggested operation.

VERIFIED REASONING

- 1 Multiply numerators: $2 \times 3 = 6$.
- 2 Multiply denominators: $3 \times 5 = 15$.
- 3 Simplify $\frac{6}{15}$ to $\frac{2}{5}$.



GUIDED PRACTICE

Multiply $\frac{3}{4} \times \frac{2}{3}$. Complete the numerator and denominator products, then simplify.

Use the visible support, then complete the mathematics.
Start with the first step shown. Complete the missing steps.

PLAN / WORK / CHECK

GUIDED PRACTICE

Multiply $\frac{5}{6} \times \frac{3}{5}$ and simplify the exact result.

Use the visible support, then complete the mathematics.
Hint: use the suggested operation.

PLAN / WORK / CHECK

SPOT THE ERROR

CLAIM TO CHECK - FIND AND REPAIR THE ERROR

A student says $\frac{2}{3} \times \frac{3}{5}$ cannot be multiplied until the denominators match. Diagnose the error and compute the product.

Inspect the claim, identify the error, and correct it.

DIAGNOSE / CORRECT / EXPLAIN



CHOOSE A METHOD

For $\frac{3}{4} \times \frac{2}{5}$, which method is valid? A) find a common denominator; B) multiply numerators and denominators, then simplify; C) add across diagonally. Justify.

Choose a method and justify why it fits the relationship.

YOUR CHOICE / WHY THIS METHOD?

INDEPENDENT PRACTICE

Multiply $\frac{7}{10} \times \frac{5}{14}$ and give the exact result in lowest terms.

Solve without a worked answer on this page.

WORK INDEPENDENTLY

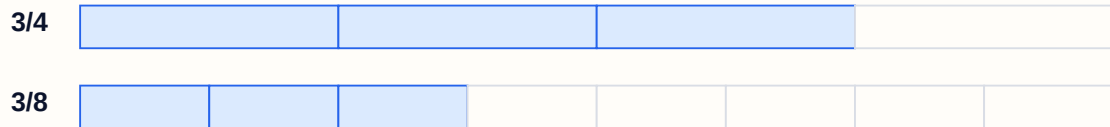


APPLY & CONNECT

The model compares $\frac{3}{4}$ with half of that amount. Write and solve the multiplication statement represented by the model.

Connect the context or model to a mathematical statement.
Use the model.

MATHEMATICAL MODEL



MODEL OR CONTEXT / MATHEMATICS / INTERPRETATION

QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, simplify $\frac{18}{24}$ to lowest terms.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, add $\frac{1}{3} + \frac{1}{6}$ and simplify.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



UNIT 11

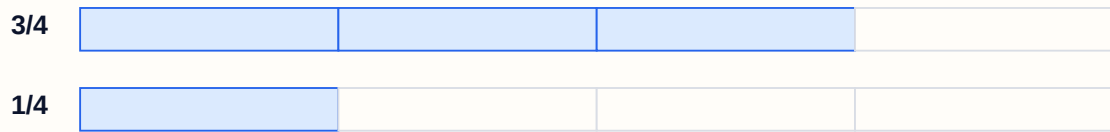
Dividing Fractions

WORKED EXAMPLE

Study the bars for $\frac{3}{4}$ and $\frac{1}{4}$. How many one-fourth units fit inside three fourths?

Study the verified reasoning before independent work.
Use the model. Use the vocabulary cue.

MATHEMATICAL MODEL



VERIFIED REASONING

- 1 Three fourths contains three groups of one fourth.
- 2 So $\frac{3}{4}$ divided by $\frac{1}{4}$ equals 3.

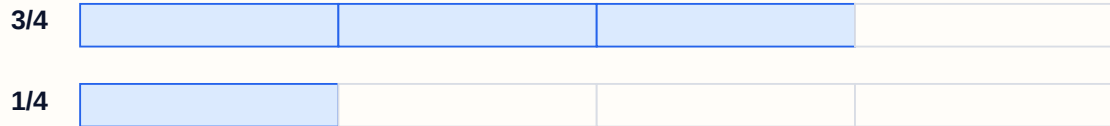


CHECK YOUR UNDERSTANDING

Using the bars, explain why $\frac{3}{4}$ divided by $\frac{1}{4}$ answers a 'how many groups?' question.

Explain the mathematical relationship, not only the result.
Use the model.

MATHEMATICAL MODEL



EXPLAIN WITH MATHEMATICAL EVIDENCE

WORKED EXAMPLE

Study the verified procedure for $\frac{2}{3}$ divided by $\frac{4}{5}$. Explain the reciprocal step before multiplying.

Study the verified reasoning before independent work.
Start with the first step shown. Use the vocabulary cue.

VERIFIED REASONING

- 1 Keep $\frac{2}{3}$ and multiply by the reciprocal of $\frac{4}{5}$, which is $\frac{5}{4}$.
- 2 $\frac{2}{3} \times \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$.



GUIDED PRACTICE

Divide $\frac{3}{5}$ by $\frac{2}{7}$. Write the reciprocal of the divisor, then multiply.

Use the visible support, then complete the mathematics.
Start with the first step shown. Complete the missing steps.

PLAN / WORK / CHECK

GUIDED PRACTICE

Divide $\frac{4}{9}$ by $\frac{2}{3}$. Choose the reciprocal and simplify the exact quotient.

Use the visible support, then complete the mathematics.
Choose from the suggested methods.

PLAN / WORK / CHECK

SPOT THE ERROR

CLAIM TO CHECK - FIND AND REPAIR THE ERROR

A student correctly rewrites $\frac{3}{4}$ divided by $\frac{1}{2}$ as $\frac{3}{4} \times \frac{2}{1}$, then says a common denominator is required before multiplying. Diagnose the error.

Inspect the claim, identify the error, and correct it.

DIAGNOSE / CORRECT / EXPLAIN



CHOOSE A METHOD

For $\frac{2}{3}$ divided by $\frac{4}{5}$, choose the valid next step: A) multiply by $\frac{4}{5}$; B) multiply by $\frac{5}{4}$; C) subtract denominators. Justify.

Choose a method and justify why it fits the relationship.

YOUR CHOICE / WHY THIS METHOD?

INDEPENDENT PRACTICE

Divide $\frac{5}{6}$ by $\frac{10}{9}$ and give the exact quotient in lowest terms.

Solve without a worked answer on this page.

WORK INDEPENDENTLY

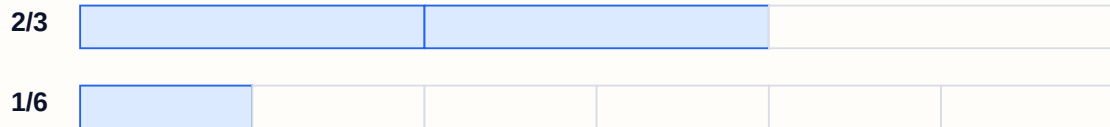


APPLY & CONNECT

Use the bars to determine how many one-sixth units fit inside two thirds, then write the matching division statement.

Connect the context or model to a mathematical statement.
Use the model.

MATHEMATICAL MODEL



MODEL OR CONTEXT / MATHEMATICS / INTERPRETATION

QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, add $\frac{2}{9} + \frac{4}{9}$ and simplify.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, subtract $\frac{5}{6} - \frac{1}{4}$.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



UNIT 12

Mixed Numbers and Improper Fractions

WORKED EXAMPLE

Study the number line from 0 to 2. What mixed number names the exact point $7/4$?

Study the verified reasoning before independent work.
Use the model. Use the vocabulary cue.

MATHEMATICAL MODEL



VERIFIED REASONING

- 1 Seven fourths contains one complete group of four fourths with three fourths left.
- 2 So $7/4 = 1 \frac{3}{4}$, and both names occupy the same point on the number line.



CHECK YOUR UNDERSTANDING

Why must $\frac{7}{4}$ and $1\frac{3}{4}$ occupy the same number-line point even though their notation is different?

Explain the mathematical relationship, not only the result.
Use the model.

MATHEMATICAL MODEL



EXPLAIN WITH MATHEMATICAL EVIDENCE

WORKED EXAMPLE

Study the conversion of $2\frac{1}{3}$ to an improper fraction. Explain why two wholes contribute six thirds before the extra third is added.

Study the verified reasoning before independent work.
Start with the first step shown. Use the vocabulary cue.

VERIFIED REASONING

- 1 Two wholes contain $2 \times 3 = 6$ thirds.
- 2 Add the remaining one third: $6 + 1 = 7$ thirds, so $2\frac{1}{3} = \frac{7}{3}$.



GUIDED PRACTICE

Convert $11\frac{1}{4}$ to a mixed number. Use quotient and remainder explicitly.

Use the visible support, then complete the mathematics.
Start with the first step shown. Complete the missing steps.

PLAN / WORK / CHECK

GUIDED PRACTICE

Convert $1\frac{5}{6}$ to an improper fraction. Account for the complete whole before adding the remaining fifths-of-sixths.

Use the visible support, then complete the mathematics.
Start with the first step shown.

PLAN / WORK / CHECK

SPOT THE ERROR

CLAIM TO CHECK - FIND AND REPAIR THE ERROR

A student says $2\frac{1}{3} = \frac{3}{3}$ by adding the whole number 2 directly to numerator 1. Diagnose the error and convert correctly.

Inspect the claim, identify the error, and correct it.

DIAGNOSE / CORRECT / EXPLAIN



INDEPENDENT PRACTICE

Convert $13/5$ to a mixed number without scaffolding.

Solve without a worked answer on this page.

WORK INDEPENDENTLY

APPLY & CONNECT

Use the number line to convert $9/4$ to a mixed number and explain how the point confirms that the value is unchanged.

Connect the context or model to a mathematical statement.
Use the model.

MATHEMATICAL MODEL



MODEL OR CONTEXT / MATHEMATICS / INTERPRETATION



QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, add $\frac{1}{4} + \frac{1}{6}$ and simplify.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY

QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, multiply $\frac{2}{3} \times \frac{3}{4}$ and simplify.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



UNIT 13

Fraction Word Problems

WORKED EXAMPLE

A tank is $\frac{3}{4}$ full. A process uses $\frac{2}{3}$ of the water currently in it. Study the model and determine what fraction of total tank capacity is used.

Study the verified reasoning before independent work.
Use the model. Hint: use the suggested operation.

MATHEMATICAL MODEL



VERIFIED REASONING

- 1 The tank contains $\frac{3}{4}$ of its capacity, and the process uses $\frac{2}{3}$ of that amount.
- 2 That relationship is multiplication: $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$.
- 3 The process uses one half of the tank's total capacity.

GUIDED PRACTICE

A trail is $\frac{5}{6}$ km long and $\frac{1}{4}$ km has been walked. Determine how much remains and state why the relationship calls for that operation.

Use the visible support, then complete the mathematics.
Hint: use the suggested operation.

PLAN / WORK / CHECK



SPOT THE ERROR

CLAIM TO CHECK - FIND AND REPAIR THE ERROR

A student says the word 'left' always means division and therefore divides when $\frac{1}{3}$ liter is poured out of $\frac{3}{4}$ liter. Diagnose the reasoning and solve the situation.

Inspect the claim, identify the error, and correct it.
Use the checklist.

DIAGNOSE / CORRECT / EXPLAIN

CHOOSE A METHOD

A quantity is two thirds of three fifths of a kilogram. Choose: A) add; B) multiply; C) divide. Select from the relationship, then verify exactly.

Choose a method and justify why it fits the relationship.
Choose from the suggested methods.

YOUR CHOICE / WHY THIS METHOD?

INDEPENDENT PRACTICE

A $\frac{3}{4}$ -meter ribbon is cut into pieces that are each $\frac{1}{8}$ meter long. Determine the number of pieces. Choose and execute the operation without a hint.

Solve without a worked answer on this page.

WORK INDEPENDENTLY

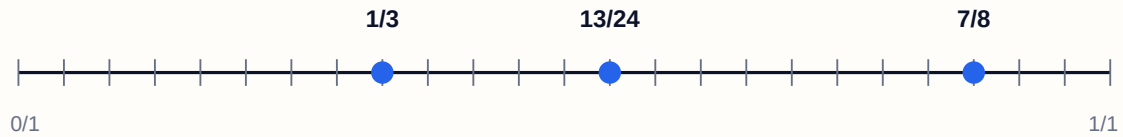


APPLY & CONNECT

A route is $\frac{7}{8}$ km long and $\frac{1}{3}$ km is removed from the distance still to cover. Use the number-line model to formulate the relationship, solve it, and interpret the remaining distance.

Connect the context or model to a mathematical statement.

MATHEMATICAL MODEL



MODEL OR CONTEXT / MATHEMATICS / INTERPRETATION

QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, simplify $\frac{20}{30}$ to lowest terms.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, subtract $\frac{1}{3}$ from $\frac{5}{6}$ and simplify.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY

QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Without looking back, compute $\frac{3}{4} \div \frac{1}{8}$.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



UNIT 14

Fractions Cumulative Review

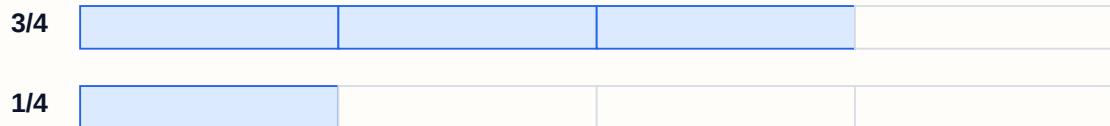
SPOT THE ERROR

CLAIM TO CHECK - FIND AND REPAIR THE ERROR

Diagnostic: a student sees the word 'after' and automatically multiplies when $\frac{1}{4}$ is poured out of $\frac{3}{4}$. Identify the reasoning error and give the exact result.

Inspect the claim, identify the error, and correct it.
Use the checklist.

MATHEMATICAL MODEL



DIAGNOSE / CORRECT / EXPLAIN

CHOOSE A METHOD

Cumulative strategy check: two thirds of three fourths is described. Choose A) add, B) multiply, or C) divide, then justify from the relationship.

Choose a method and justify why it fits the relationship.
Choose from the suggested methods.

YOUR CHOICE / WHY THIS METHOD?



INDEPENDENT PRACTICE

Complete this four-part cumulative set without hints: compare $\frac{5}{8}$ and $\frac{2}{3}$; add $\frac{1}{3} + \frac{1}{4}$; divide $\frac{3}{4}$ by $\frac{1}{8}$; convert $1\frac{1}{4}$ to a mixed number.

Solve without a worked answer on this page.

WORK INDEPENDENTLY

APPLY & CONNECT

Use the number line only as a model: convert $\frac{7}{4}$ to a mixed number and explain why both notations identify the same point.

Connect the context or model to a mathematical statement.
Use the model.

MATHEMATICAL MODEL



MODEL OR CONTEXT / MATHEMATICS / INTERPRETATION



QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Retrieve the original meaning of $\frac{3}{5}$: state what it means relative to one whole.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY

QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Represent $\frac{2}{5}$ as equal parts of one whole without looking back.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY

QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Decide whether $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent and justify exactly.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Simplify $18/24$ to lowest terms.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY

QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Add $1/3 + 1/6$ and simplify.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY

QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Multiply $2/3 \times 3/4$ and simplify.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



QUICK RECALL

TRY THIS FROM MEMORY - DO NOT LOOK BACK

Convert $11\frac{1}{4}$ to a mixed number.

Recall earlier learning without looking back at the source unit.

RECALL AND SOLVE FROM MEMORY



Verified Solutions

Use this appendix after attempting learner-response activities.

Worked examples already reveal their reasoning in the learning sequence.

Every solution below remains bound to the certified deterministic truth record.



WORKED EXAMPLE

UNIT 01

Study the bar. What does $\frac{3}{4}$ mean when the whole is divided into equal parts?

1. The whole is divided into 4 equal parts.
2. Three of those equal parts are selected, so the fraction is $\frac{3}{4}$.

Verified result: $\frac{3}{4}$

CHECK YOUR UNDERSTANDING

UNIT 01

Using the bar, explain what the 3 and the 4 each tell us in $\frac{3}{4}$.

1. The denominator 4 tells how many equal parts make one whole.
2. The numerator 3 tells how many of those parts are selected.

Verified result: $\frac{3}{4}$

GUIDED PRACTICE

UNIT 01

A whole is divided into 5 equal parts and 2 are selected. Write the fraction.

1. Five equal parts make the whole and two are selected.
2. Write $\frac{2}{5}$.

Verified result: $\frac{2}{5}$

INDEPENDENT PRACTICE

UNIT 01

A whole has 7 equal parts and 4 are selected. Write the fraction without a hint.

1. Seven equal parts make the whole and four are selected.
2. Write $\frac{4}{7}$.

Verified result: $\frac{4}{7}$



APPLY & CONNECT

UNIT 01

The point on the number line is $\frac{3}{4}$. Explain how that location expresses the same equal-parts meaning as a fraction bar.

1. Three quarters is the number $\frac{3}{4}$.
2. The marked point lies three of four equal quarter-steps from zero to one.

Verified result: $\frac{3}{4}$

WORKED EXAMPLE

UNIT 02

Study $\frac{3}{5}$ and the bar. Identify the numerator and denominator and state each role.

1. In $\frac{3}{5}$, 3 is the numerator and 5 is the denominator.
2. The denominator sets the equal-part size; the numerator counts selected parts.

Verified result: $\frac{3}{5}$

CHECK YOUR UNDERSTANDING

UNIT 02

Why does the denominator 5 in $\frac{3}{5}$ not mean that five pieces are selected?

1. The 5 means the whole is partitioned into five equal parts.
2. The 3 means three of those parts are selected.

Verified result: $\frac{3}{5}$

SPOT THE ERROR

UNIT 02

A student says: 'In $\frac{3}{5}$, both 3 and 5 count selected pieces.' What is wrong with that explanation?

1. The claim is incorrect because numerator and denominator have different roles.
2. Only the numerator counts selected equal parts; the denominator defines how many equal parts make the whole.

Verified result: INCORRECT



GUIDED PRACTICE

UNIT 02

For $\frac{4}{7}$, label numerator and denominator, then finish: the denominator tells ____; the numerator tells ____.

1. 4 is the numerator: selected parts.
2. 7 is the denominator: equal parts in the whole.

Verified result: $\frac{4}{7}$

INDEPENDENT PRACTICE

UNIT 02

Explain the roles of 5 and 8 in $\frac{5}{8}$ without using a hint.

1. 5 is the numerator and 8 is the denominator.
2. The fraction names five selected eighths of one whole.

Verified result: $\frac{5}{8}$

WORKED EXAMPLE

UNIT 03

Study the bar and connect its five equal parts and two shaded parts to the symbol $\frac{2}{5}$.

1. The bar has five equal parts and two are selected.
2. That model represents $\frac{2}{5}$.

Verified result: $\frac{2}{5}$

CHECK YOUR UNDERSTANDING

UNIT 03

Why does this bar represent $\frac{2}{5}$ rather than $\frac{5}{2}$?

1. The denominator 5 fixes fifth-sized parts and the numerator 2 selects two of them.
2. So the bar and the symbol name the same fraction.

Verified result: $\frac{2}{5}$



GUIDED PRACTICE

UNIT 03

A model is divided into 8 equal parts and 3 are selected. Complete the symbol: $\frac{\quad}{\quad}$.

1. Eight equal parts make the whole and three are selected.
2. The matching symbol is $\frac{3}{8}$.

Verified result: $\frac{3}{8}$

INDEPENDENT PRACTICE

UNIT 03

Write the fraction represented by a whole divided into 5 equal parts with 2 selected.

1. A five-part whole with two selected parts is represented by $\frac{2}{5}$.

Verified result: $\frac{2}{5}$

APPLY & CONNECT

UNIT 03

The number line marks $\frac{2}{5}$. Explain why that point and a bar with two of five equal parts shaded represent the same number.

1. The point is two fifth-steps from zero on a zero-to-one scale.
2. It therefore represents the same number $\frac{2}{5}$ as the bar.

Verified result: $\frac{2}{5}$

QUICK RECALL

UNIT 03

Without looking back: in $\frac{3}{7}$, what does the denominator 7 mean and what does the numerator 3 mean?

1. In $\frac{3}{7}$, the denominator 7 says the whole is divided into seven equal parts.
2. The numerator 3 says three of those equal parts are selected.

Verified result: $\frac{3}{7}$



WORKED EXAMPLE

UNIT 04

Study the two equal-length bars. Which is greater, $\frac{2}{3}$ or $\frac{2}{5}$, and what does the picture show?

1. Both fractions use the same-sized whole.
2. Two thirds covers more of that whole than two fifths, so $\frac{2}{3} > \frac{2}{5}$.

Verified result: $\frac{2}{3} > \frac{2}{5}$

CHECK YOUR UNDERSTANDING

UNIT 04

Use the number line to explain why $\frac{2}{3}$ is greater than $\frac{2}{5}$.

1. On one number line, $\frac{2}{3}$ lies to the right of $\frac{2}{5}$.
2. Therefore $\frac{2}{3} > \frac{2}{5}$.

Verified result: $\frac{2}{3} > \frac{2}{5}$

GUIDED PRACTICE

UNIT 04

Compare $\frac{3}{4}$ and $\frac{2}{3}$. Use a common scale or cross-products, then write $>$, $<$, or $=$.

1. Use a common scale or exact cross-products: $3 \times 3 = 9$ and $2 \times 4 = 8$.
2. Since $9 > 8$, $\frac{3}{4} > \frac{2}{3}$.

Verified result: $\frac{3}{4} > \frac{2}{3}$

INDEPENDENT PRACTICE

UNIT 04

Compare $\frac{5}{8}$ and $\frac{3}{4}$ without a hint. Write the correct relation and justify it exactly.

1. Exact comparison gives $\frac{5}{8} < \frac{3}{4}$.
2. For example, $5 \times 4 = 20$ while $3 \times 8 = 24$.

Verified result: $\frac{5}{8} < \frac{3}{4}$



SPOT THE ERROR

UNIT 04

A student says $2/3 < 2/5$ because 5 is the larger denominator. Diagnose the error.

1. The claim is incorrect.
2. With equal numerators and the same whole, more denominator parts means each part is smaller; exact comparison gives $2/3 > 2/5$.

Verified result: **INCORRECT**

CHOOSE A METHOD

UNIT 04

To compare $3/4$ and $2/3$ reliably, choose: A) compare denominators only; B) compare numerators only; C) use a common exact scale or cross-products. Explain why.

1. Choose C: place both fractions on a common exact scale or compare cross-products.
2. Comparing only numerators or only denominators does not generally determine fraction size.

Verified result: **C**

APPLY & CONNECT

UNIT 04

The bars show $3/4$ and $2/3$ of equal wholes. Which fraction is larger? Connect the visual model to an exact symbolic comparison.

1. The three-quarter bar covers more of the same-sized whole than the two-thirds bar.
2. Exact comparison confirms $3/4 > 2/3$.

Verified result: **$3/4 > 2/3$**

QUICK RECALL

UNIT 04

Without looking back: in $5/9$, identify numerator and denominator and state their different roles.

1. In $5/9$, 9 is the denominator and tells how many equal parts make the whole.
2. 5 is the numerator and tells how many parts are selected.

Verified result: **$5/9$**



WORKED EXAMPLE

UNIT 05

Study the two fraction bars. They look partitioned differently. What relationship do they show?

1. Both bars cover exactly one half of the same-sized whole.
2. Different numbers of equal parts can represent the same value.
3. Therefore $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.

Verified result: $\frac{1}{2} = \frac{1}{2}$

CHECK YOUR UNDERSTANDING

UNIT 05

Do $\frac{1}{2}$ and $\frac{2}{4}$ represent the same amount? Explain using the bars.

1. The shaded amount is the same in both bars.
2. The first names that amount with 2 equal parts; the second names it with 4 equal parts.

Verified result: SAME_VALUE

CHECK YOUR UNDERSTANDING

UNIT 05

On the number line, should $\frac{1}{2}$ and $\frac{2}{4}$ appear at different points or the same point? Why?

1. Equivalent fractions name the same number.
2. So $\frac{1}{2}$ and $\frac{2}{4}$ must land at the same point on the number line.

Verified result: SAME_POINT

GUIDED PRACTICE

UNIT 05

Complete the equivalent fraction: $\frac{3}{5} = \frac{\quad}{10}$. What factor changes 5 into 10?

1. Multiply the numerator and denominator by the same nonzero factor.
2. $3 \times 2 = 6$ and $5 \times 2 = 10$.
3. So $\frac{3}{5} = \frac{6}{10}$.

Verified result: $\frac{6}{10}$



INDEPENDENT PRACTICE

UNIT 05

Write $\frac{4}{7}$ as an equivalent fraction with denominator 21.

1. To reach denominator 21 from 7, multiply by 3.
2. Multiply the numerator by the same factor: $4 \times 3 = 12$.
3. So $\frac{4}{7} = \frac{12}{21}$.

Verified result: **$\frac{12}{21}$**

SPOT THE ERROR

UNIT 05

A student says $\frac{3}{4} = \frac{4}{5}$ because adding 1 to both numerator and denominator preserves equivalence. Diagnose the error and correct the claim.

1. The claim is incorrect: adding the same number to numerator and denominator does not generally preserve a fraction's value.
2. Exact comparison gives $\frac{3}{4} < \frac{4}{5}$, so the two fractions are not equivalent.
3. To generate an equivalent fraction, multiply or divide numerator and denominator by the same nonzero factor.

Verified result: **INCORRECT**

CHOOSE A METHOD

UNIT 05

Which method keeps $\frac{3}{4}$ equivalent? A) add 1 to numerator and denominator; B) multiply both by 2; C) multiply only the numerator by 2.

1. Option B is the only method that preserves value.
2. Multiplying both numerator and denominator by 2 gives $\frac{6}{8}$, which equals $\frac{3}{4}$.

Verified result: **B**

APPLY & CONNECT

UNIT 05

The model shows $\frac{3}{6}$ of a whole. Which simpler fraction names the same amount?

1. Three of six equal parts is $\frac{3}{6}$.
2. Dividing numerator and denominator by 3 gives $\frac{1}{2}$.
3. The model therefore represents one half.

Verified result: **$\frac{1}{2}$**



QUICK RECALL

UNIT 05

Without looking back, represent $\frac{3}{5}$ as equal parts of one whole and state how many of those parts are selected.

1. Divide one whole into five equal parts.
2. Select three of those five equal parts.
3. That representation is $\frac{3}{5}$.

Verified result: $\frac{3}{5}$

WORKED EXAMPLE

UNIT 06

Study how $\frac{6}{8}$ becomes $\frac{3}{4}$ without changing the shaded amount.

1. The greatest common divisor of 6 and 8 is 2.
2. Divide numerator and denominator by the same nonzero factor: $(6 \div 2)/(8 \div 2) = \frac{3}{4}$.
3. The visual model confirms that the value did not change.

Verified result: $\frac{3}{4}$

GUIDED PRACTICE

UNIT 06

Simplify $\frac{12}{18}$. First identify a common factor, then divide numerator and denominator by it.

1. A common factor is 6.
2. $(12 \div 6)/(18 \div 6) = \frac{2}{3}$.
3. $\frac{2}{3}$ is in lowest terms.

Verified result: $\frac{2}{3}$

GUIDED PRACTICE

UNIT 06

Simplify $\frac{15}{20}$ to lowest terms. Use the greatest common factor if you can find it.

1. The greatest common divisor of 15 and 20 is 5.
2. $(15 \div 5)/(20 \div 5) = \frac{3}{4}$.

Verified result: $\frac{3}{4}$



INDEPENDENT PRACTICE

UNIT 06

Simplify $21/28$ to lowest terms.

1. The greatest common divisor of 21 and 28 is 7.
2. $(21 \div 7)/(28 \div 7) = 3/4$.

Verified result: $3/4$

INDEPENDENT PRACTICE

UNIT 06

Simplify $36/48$ to lowest terms.

1. The greatest common divisor of 36 and 48 is 12.
2. $(36 \div 12)/(48 \div 12) = 3/4$.

Verified result: $3/4$

SPOT THE ERROR

UNIT 06

A student says $6/8$ simplifies to $3/8$ because only the numerator needs to be divided by 2. Diagnose the error and give the correct lowest terms.

1. The claim is incorrect because changing only the numerator changes the fraction's value.
2. To preserve value, divide numerator and denominator by the same nonzero factor: $(6 \div 2)/(8 \div 2) = 3/4$.
3. So $6/8$ simplifies to $3/4$, not $3/8$.

Verified result: **INCORRECT**

CHOOSE A METHOD

UNIT 06

To simplify $18/24$ to lowest terms in one step, which common factor should you divide by? A) 2, B) 3, C) 6.

1. Choose C: 6.
2. 6 is the greatest common divisor of 18 and 24, so dividing both by 6 reaches $3/4$ in one step.

Verified result: **C**



APPLY & CONNECT

UNIT 06

The bar model represents $\frac{12}{16}$ of a whole. Write the same amount as a fraction in lowest terms.

1. The model represents $\frac{12}{16}$.
2. Divide numerator and denominator by 4: $(12 \div 4) / (16 \div 4) = \frac{3}{4}$.
3. So the same modeled quantity is $\frac{3}{4}$ in lowest terms.

Verified result: $\frac{3}{4}$

QUICK RECALL

UNIT 06

Without looking back, compare $\frac{3}{5}$ and $\frac{2}{3}$ on a common scale and state the exact relation.

1. Use fifteenths as a common scale: $\frac{3}{5} = \frac{9}{15}$ and $\frac{2}{3} = \frac{10}{15}$.
2. Because $\frac{9}{15} < \frac{10}{15}$, $\frac{3}{5} < \frac{2}{3}$.

Verified result: $\frac{3}{5} < \frac{2}{3}$

WORKED EXAMPLE

UNIT 07

Study the seventh-bars for $\frac{2}{7}$, $\frac{3}{7}$, and $\frac{5}{7}$. What stays unchanged when the two fractions are added?

1. Both fractions use sevenths, so the pieces already have the same size.
2. Combine the selected pieces: $2 + 3 = 5$ while the denominator remains 7.
3. Therefore $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$.

Verified result: $\frac{5}{7}$

CHECK YOUR UNDERSTANDING

UNIT 07

Why does the denominator remain 7 in $\frac{2}{7} + \frac{3}{7}$? Explain using equal-sized parts.

1. The denominator names the size of each equal part.
2. Adding like denominators changes how many sevenths are selected, not the size of a seventh.

Verified result: $\frac{5}{7}$



WORKED EXAMPLE

UNIT 07

Study the worked reasoning for $\frac{1}{8} + \frac{4}{8}$. Why can the numerators be combined immediately?

1. Model the procedure: keep denominator 8 and add numerators $1 + 4$.
2. The result is $\frac{5}{8}$.

Verified result: $\frac{5}{8}$

GUIDED PRACTICE

UNIT 07

Add $\frac{3}{9} + \frac{2}{9}$. Use the common denominator and complete the numerator step.

1. The common denominator is already 9.
2. Add $3 + 2$ and keep 9: $\frac{5}{9}$.

Verified result: $\frac{5}{9}$

GUIDED PRACTICE

UNIT 07

Add $\frac{1}{6} + \frac{4}{6}$. Decide what changes and what remains fixed.

1. The common denominator is already 6.
2. Add $1 + 4$ and keep 6: $\frac{5}{6}$.

Verified result: $\frac{5}{6}$

SPOT THE ERROR

UNIT 07

A student writes $\frac{2}{7} + \frac{3}{7} = \frac{5}{14}$. Identify the error and give the correct sum.

1. The claim is incorrect.
2. Sevenths remain sevenths; adding the denominators would change the part size.
3. The verified sum is $\frac{5}{7}$, not $\frac{5}{14}$.

Verified result: **INCORRECT**



INDEPENDENT PRACTICE

UNIT 07

Add $5/12 + 4/12$ and give the final answer in lowest terms.

1. Add the numerators: $5 + 4 = 9$, giving $9/12$.
2. Simplify $9/12$ to $3/4$.

Verified result: $3/4$

APPLY & CONNECT

UNIT 07

The model shows the result of $2/8 + 3/8$. Connect the symbolic sum to the shaded amount and state the result.

1. $2/8 + 3/8 = 5/8$.
2. The bar shows five eighths, matching the exact sum.

Verified result: $5/8$

QUICK RECALL

UNIT 07

Without looking back, write $3/4$ as an equivalent fraction with denominator 12.

1. Multiply numerator and denominator of $3/4$ by 3.
2. $3/4 = 9/12$.

Verified result: $9/12$

WORKED EXAMPLE

UNIT 08

Study the $1/2$ and $1/3$ bars. Why can their numerators not be combined immediately?

1. Halves and thirds are different-sized parts, so their numerators cannot be combined yet.
2. Rewrite them on a common scale: $1/2 = 3/6$ and $1/3 = 2/6$.
3. Then $3/6 + 2/6 = 5/6$.

Verified result: $5/6$



CHECK YOUR UNDERSTANDING

UNIT 08

Explain why a common denominator is about equal-sized parts, not merely matching symbols.

1. The bars use different partition sizes.
2. A common denominator makes the parts equal-sized before their counts are combined.

Verified result: $\frac{5}{6}$

WORKED EXAMPLE

UNIT 08

Study the worked addition $\frac{1}{3} + \frac{1}{4}$. Follow the equivalent-fraction conversion before adding.

1. Use denominator 12: $\frac{1}{3} = \frac{4}{12}$ and $\frac{1}{4} = \frac{3}{12}$.
2. Then $\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$.

Verified result: $\frac{7}{12}$

GUIDED PRACTICE

UNIT 08

Add $\frac{1}{2} + \frac{1}{6}$. First rewrite the half using sixths.

1. Use denominator 6: $\frac{1}{2} = \frac{3}{6}$.
2. $\frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$.

Verified result: $\frac{2}{3}$

GUIDED PRACTICE

UNIT 08

Add $\frac{2}{5} + \frac{1}{10}$. Choose a common denominator and complete the sum.

1. Use denominator 10: $\frac{2}{5} = \frac{4}{10}$.
2. $\frac{4}{10} + \frac{1}{10} = \frac{5}{10} = \frac{1}{2}$.

Verified result: $\frac{1}{2}$



SPOT THE ERROR

UNIT 08

A student says $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$. Diagnose the error and repair the calculation.

1. The claim is incorrect because halves and thirds are not equal-sized parts.
2. Convert to sixths first: $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.

Verified result: **INCORRECT**

CHOOSE A METHOD

UNIT 08

For $\frac{1}{3} + \frac{1}{4}$, which proposed common denominator works? A) 7, B) 12, C) 20. Justify the choice.

1. Choose B: 12.
2. Twelve is a common multiple of both denominators and preserves equivalent values.

Verified result: **B**

INDEPENDENT PRACTICE

UNIT 08

Add $\frac{3}{8} + \frac{1}{6}$ and give the exact result in lowest terms.

1. The least common denominator is 24.
2. $\frac{3}{8} = \frac{9}{24}$ and $\frac{1}{6} = \frac{4}{24}$, so the sum is $\frac{13}{24}$.

Verified result: **$\frac{13}{24}$**

APPLY & CONNECT

UNIT 08

Use the bars to connect $\frac{1}{2} + \frac{1}{3}$ with the final amount $\frac{5}{6}$. Explain the common-scale step.

1. Convert $\frac{1}{2}$ to $\frac{3}{6}$ and $\frac{1}{3}$ to $\frac{2}{6}$.
2. Their sum is $\frac{5}{6}$, matching the result model.

Verified result: **$\frac{5}{6}$**



QUICK RECALL

UNIT 08

Without looking back, simplify $12/16$ to lowest terms.

1. The greatest common divisor of 12 and 16 is 4.
2. Divide both by 4: $12/16 = 3/4$.

Verified result: $3/4$

WORKED EXAMPLE

UNIT 09

Study $3/4 - 1/4$ with the bars. What happens to the selected fourths, and how does the result simplify?

1. Both fractions use fourths, so subtract the selected fourths.
2. $3/4 - 1/4 = 2/4 = 1/2$.

Verified result: $1/2$

GUIDED PRACTICE

UNIT 09

Subtract $5/6 - 1/3$. First rewrite the third using sixths.

1. Rewrite $1/3$ as $2/6$.
2. $5/6 - 2/6 = 3/6 = 1/2$.

Verified result: $1/2$

GUIDED PRACTICE

UNIT 09

Subtract $7/8 - 1/4$. Choose an equivalent form for $1/4$ before subtracting.

1. Rewrite $1/4$ as $2/8$.
2. $7/8 - 2/8 = 5/8$.

Verified result: $5/8$



CHOOSE A METHOD

UNIT 09

For $\frac{3}{4} - \frac{1}{6}$, which listed common denominator is the best choice? A) 10, B) 12, C) 24. Justify.

1. Choose B: 12.
2. Twelve is divisible by both 4 and 6 and lets both fractions use equal-sized parts.

Verified result: B

SPOT THE ERROR

UNIT 09

A student writes $\frac{3}{4} - \frac{1}{2} = \frac{2}{2}$. Diagnose the error and calculate the exact difference.

1. The claim is incorrect; numerator and denominator cannot be subtracted independently.
2. Rewrite $\frac{1}{2}$ as $\frac{2}{4}$, then $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$.

Verified result: INCORRECT

INDEPENDENT PRACTICE

UNIT 09

Subtract $\frac{11}{12} - \frac{2}{9}$ and give the exact result in lowest terms.

1. Use denominator 36: $\frac{11}{12} = \frac{33}{36}$ and $\frac{2}{9} = \frac{8}{36}$.
2. $\frac{33}{36} - \frac{8}{36} = \frac{25}{36}$.

Verified result: $\frac{25}{36}$

APPLY & CONNECT

UNIT 09

Use the model to explain why $\frac{5}{6} - \frac{1}{3}$ equals $\frac{1}{2}$ after equivalent conversion and simplification.

1. $\frac{1}{3} = \frac{2}{6}$, so $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$.
2. Simplify $\frac{3}{6}$ to $\frac{1}{2}$, which matches the model.

Verified result: $\frac{1}{2}$



QUICK RECALL

UNIT 09

Without looking back, write $\frac{2}{3}$ as an equivalent fraction with denominator 12.

1. Multiply numerator and denominator of $\frac{2}{3}$ by 4.
2. $\frac{2}{3} = \frac{8}{12}$.

Verified result: $\frac{8}{12}$

QUICK RECALL

UNIT 09

Without looking back, add $\frac{3}{10} + \frac{4}{10}$.

1. Tenths already have the same part size.
2. $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$.

Verified result: $\frac{7}{10}$

WORKED EXAMPLE

UNIT 10

Study the bars for $\frac{2}{3}$ and $\frac{1}{3}$. Why does taking one half of $\frac{2}{3}$ produce $\frac{1}{3}$?

1. Two thirds of the whole is the starting amount.
2. Taking one half of that amount leaves one third of the whole.
3. Exactly: $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$.

Verified result: $\frac{1}{3}$

CHECK YOUR UNDERSTANDING

UNIT 10

Using the bars, explain why the word 'of' in one half of two thirds can be represented by multiplication.

1. The second bar is half as much as the two-thirds bar.
2. That relationship is multiplication by $\frac{1}{2}$, so the exact result is $\frac{1}{3}$.

Verified result: $\frac{1}{3}$



WORKED EXAMPLE

UNIT 10

Study the verified procedure for $\frac{2}{3} \times \frac{3}{5}$. Track the unsimplified product before reducing it.

1. Multiply numerators: $2 \times 3 = 6$.
2. Multiply denominators: $3 \times 5 = 15$.
3. Simplify $\frac{6}{15}$ to $\frac{2}{5}$.

Verified result: $\frac{2}{5}$

GUIDED PRACTICE

UNIT 10

Multiply $\frac{3}{4} \times \frac{2}{3}$. Complete the numerator and denominator products, then simplify.

1. Multiply 3×2 over 4×3 to obtain $\frac{6}{12}$.
2. Simplify $\frac{6}{12}$ to $\frac{1}{2}$.

Verified result: $\frac{1}{2}$

GUIDED PRACTICE

UNIT 10

Multiply $\frac{5}{6} \times \frac{3}{5}$ and simplify the exact result.

1. Multiply 5×3 over 6×5 to obtain $\frac{15}{30}$.
2. Simplify to $\frac{1}{2}$.

Verified result: $\frac{1}{2}$

SPOT THE ERROR

UNIT 10

A student says $\frac{2}{3} \times \frac{3}{5}$ cannot be multiplied until the denominators match. Diagnose the error and compute the product.

1. The claim is incorrect.
2. A common denominator is not required for multiplication.
3. $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$.

Verified result: INCORRECT



CHOOSE A METHOD

UNIT 10

For $\frac{3}{4} \times \frac{2}{5}$, which method is valid? A) find a common denominator; B) multiply numerators and denominators, then simplify; C) add across diagonally. Justify.

1. Choose B.
2. $\frac{3}{4} \times \frac{2}{5} = \frac{6}{20} = \frac{3}{10}$; a common denominator is unnecessary.

Verified result: B

INDEPENDENT PRACTICE

UNIT 10

Multiply $\frac{7}{10} \times \frac{5}{14}$ and give the exact result in lowest terms.

1. $\frac{7}{10} \times \frac{5}{14} = \frac{35}{140}$.
2. Simplify $\frac{35}{140}$ to $\frac{1}{4}$.

Verified result: $\frac{1}{4}$

APPLY & CONNECT

UNIT 10

The model compares $\frac{3}{4}$ with half of that amount. Write and solve the multiplication statement represented by the model.

1. One half of three fourths is $\frac{3}{8}$.
2. The exact symbolic product $\frac{1}{2} \times \frac{3}{4}$ gives the same value.

Verified result: $\frac{3}{8}$

QUICK RECALL

UNIT 10

Without looking back, simplify $\frac{18}{24}$ to lowest terms.

1. The greatest common divisor of 18 and 24 is 6.
2. $\frac{18}{24} = \frac{3}{4}$.

Verified result: $\frac{3}{4}$



QUICK RECALL

UNIT 10

Without looking back, add $\frac{1}{3} + \frac{1}{6}$ and simplify.

1. Use sixths: $\frac{1}{3} = \frac{2}{6}$.
2. $\frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$.

Verified result: $\frac{1}{2}$

WORKED EXAMPLE

UNIT 11

Study the bars for $\frac{3}{4}$ and $\frac{1}{4}$. How many one-fourth units fit inside three fourths?

1. Three fourths contains three groups of one fourth.
2. So $\frac{3}{4}$ divided by $\frac{1}{4}$ equals 3.

Verified result: 3

CHECK YOUR UNDERSTANDING

UNIT 11

Using the bars, explain why $\frac{3}{4}$ divided by $\frac{1}{4}$ answers a 'how many groups?' question.

1. Division asks how many copies of the divisor fit in the dividend.
2. Exactly three one-fourth pieces fit in three fourths.

Verified result: 3

WORKED EXAMPLE

UNIT 11

Study the verified procedure for $\frac{2}{3}$ divided by $\frac{4}{5}$. Explain the reciprocal step before multiplying.

1. Keep $\frac{2}{3}$ and multiply by the reciprocal of $\frac{4}{5}$, which is $\frac{5}{4}$.
2. $\frac{2}{3} \times \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$.

Verified result: $\frac{5}{6}$



GUIDED PRACTICE

UNIT 11

Divide $\frac{3}{5}$ by $\frac{2}{7}$. Write the reciprocal of the divisor, then multiply.

1. Replace division by $\frac{2}{7}$ with multiplication by $\frac{7}{2}$.
2. $\frac{3}{5} \times \frac{7}{2} = \frac{21}{10}$.

Verified result: $\frac{21}{10}$

GUIDED PRACTICE

UNIT 11

Divide $\frac{4}{9}$ by $\frac{2}{3}$. Choose the reciprocal and simplify the exact quotient.

1. Replace division by $\frac{2}{3}$ with multiplication by $\frac{3}{2}$.
2. $\frac{4}{9} \times \frac{3}{2} = \frac{12}{18} = \frac{2}{3}$.

Verified result: $\frac{2}{3}$

SPOT THE ERROR

UNIT 11

A student correctly rewrites $\frac{3}{4}$ divided by $\frac{1}{2}$ as $\frac{3}{4} \times \frac{2}{1}$, then says a common denominator is required before multiplying. Diagnose the error.

1. The claim is incorrect.
2. Once division becomes $\frac{3}{4} \times \frac{2}{1}$, multiplication does not require a common denominator.
3. The exact quotient is $\frac{3}{2}$.

Verified result: INCORRECT

CHOOSE A METHOD

UNIT 11

For $\frac{2}{3}$ divided by $\frac{4}{5}$, choose the valid next step: A) multiply by $\frac{4}{5}$; B) multiply by $\frac{5}{4}$; C) subtract denominators. Justify.

1. Choose B.
2. $\frac{2}{3}$ divided by $\frac{4}{5}$ becomes $\frac{2}{3} \times \frac{5}{4} = \frac{5}{6}$.

Verified result: B



INDEPENDENT PRACTICE

UNIT 11

Divide $\frac{5}{6}$ by $\frac{10}{9}$ and give the exact quotient in lowest terms.

1. Multiply $\frac{5}{6}$ by the reciprocal $\frac{9}{10}$.
2. $\frac{45}{60}$ simplifies to $\frac{3}{4}$.

Verified result: $\frac{3}{4}$

APPLY & CONNECT

UNIT 11

Use the bars to determine how many one-sixth units fit inside two thirds, then write the matching division statement.

1. Two thirds is four sixths.
2. Therefore four groups of $\frac{1}{6}$ fit in $\frac{2}{3}$, so the quotient is 4.

Verified result: 4

QUICK RECALL

UNIT 11

Without looking back, add $\frac{2}{9} + \frac{4}{9}$ and simplify.

1. Nines already have equal part size.
2. $\frac{2}{9} + \frac{4}{9} = \frac{6}{9} = \frac{2}{3}$.

Verified result: $\frac{2}{3}$

QUICK RECALL

UNIT 11

Without looking back, subtract $\frac{5}{6} - \frac{1}{4}$.

1. Use twelfths: $\frac{5}{6} = \frac{10}{12}$ and $\frac{1}{4} = \frac{3}{12}$.
2. $\frac{10}{12} - \frac{3}{12} = \frac{7}{12}$.

Verified result: $\frac{7}{12}$



WORKED EXAMPLE

UNIT 12

Study the number line from 0 to 2. What mixed number names the exact point $\frac{7}{4}$?

1. Seven fourths contains one complete group of four fourths with three fourths left.
2. So $\frac{7}{4} = 1\frac{3}{4}$, and both names occupy the same point on the number line.

Verified result: $1\frac{3}{4}$

CHECK YOUR UNDERSTANDING

UNIT 12

Why must $\frac{7}{4}$ and $1\frac{3}{4}$ occupy the same number-line point even though their notation is different?

1. Converting the notation does not change the quantity.
2. $\frac{7}{4}$ and $1\frac{3}{4}$ name the same point between one and two.

Verified result: $1\frac{3}{4}$

WORKED EXAMPLE

UNIT 12

Study the conversion of $2\frac{1}{3}$ to an improper fraction. Explain why two wholes contribute six thirds before the extra third is added.

1. Two wholes contain $2 \times 3 = 6$ thirds.
2. Add the remaining one third: $6 + 1 = 7$ thirds, so $2\frac{1}{3} = \frac{7}{3}$.

Verified result: $\frac{7}{3}$

GUIDED PRACTICE

UNIT 12

Convert $\frac{11}{4}$ to a mixed number. Use quotient and remainder explicitly.

1. 11 divided by 4 is 2 remainder 3.
2. Therefore $\frac{11}{4} = 2\frac{3}{4}$.

Verified result: $2\frac{3}{4}$



GUIDED PRACTICE

UNIT 12

Convert $1\frac{5}{6}$ to an improper fraction. Account for the complete whole before adding the remaining fifths-of-sixths.

1. One whole is $\frac{6}{6}$; add the remaining $\frac{5}{6}$.
2. $1\frac{5}{6} = \frac{11}{6}$.

Verified result: $\frac{11}{6}$

SPOT THE ERROR

UNIT 12

A student says $2\frac{1}{3} = \frac{3}{3}$ by adding the whole number 2 directly to numerator 1. Diagnose the error and convert correctly.

1. The claim is incorrect because the whole number represents two complete groups of thirds.
2. 2 wholes = $\frac{6}{3}$, then $\frac{6}{3} + \frac{1}{3} = \frac{7}{3}$.

Verified result: INCORRECT

INDEPENDENT PRACTICE

UNIT 12

Convert $\frac{13}{5}$ to a mixed number without scaffolding.

1. 13 divided by 5 is 2 remainder 3.
2. So $\frac{13}{5} = 2\frac{3}{5}$.

Verified result: $2\frac{3}{5}$

APPLY & CONNECT

UNIT 12

Use the number line to convert $\frac{9}{4}$ to a mixed number and explain how the point confirms that the value is unchanged.

1. 9 divided by 4 is 2 remainder 1.
2. So $\frac{9}{4} = 2\frac{1}{4}$, matching the marked point on the number line.

Verified result: $2\frac{1}{4}$



QUICK RECALL

UNIT 12

Without looking back, add $\frac{1}{4} + \frac{1}{6}$ and simplify.

1. Use twelfths: $\frac{1}{4} = \frac{3}{12}$ and $\frac{1}{6} = \frac{2}{12}$.
2. The sum is $\frac{5}{12}$.

Verified result: $\frac{5}{12}$

QUICK RECALL

UNIT 12

Without looking back, multiply $\frac{2}{3} \times \frac{3}{4}$ and simplify.

1. Multiply 2 x 3 over 3 x 4 to obtain $\frac{6}{12}$.
2. Simplify to $\frac{1}{2}$.

Verified result: $\frac{1}{2}$

WORKED EXAMPLE

UNIT 13

A tank is $\frac{3}{4}$ full. A process uses $\frac{2}{3}$ of the water currently in it. Study the model and determine what fraction of total tank capacity is used.

1. The tank contains $\frac{3}{4}$ of its capacity, and the process uses $\frac{2}{3}$ of that amount.
2. That relationship is multiplication: $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$.
3. The process uses one half of the tank's total capacity.

Verified result: $\frac{1}{2}$

GUIDED PRACTICE

UNIT 13

A trail is $\frac{5}{6}$ km long and $\frac{1}{4}$ km has been walked. Determine how much remains and state why the relationship calls for that operation.

1. The whole trail length is $\frac{5}{6}$ km and $\frac{1}{4}$ km has already been covered.
2. Remaining distance is subtraction: $\frac{5}{6} - \frac{1}{4} = \frac{10}{12} - \frac{3}{12} = \frac{7}{12}$ km.

Verified result: $\frac{7}{12}$



SPOT THE ERROR

UNIT 13

A student says the word 'left' always means division and therefore divides when $\frac{1}{3}$ liter is poured out of $\frac{3}{4}$ liter. Diagnose the reasoning and solve the situation.

1. The claim is incorrect because an isolated word does not determine the operation.
2. The relationship removes $\frac{1}{3}$ liter from $\frac{3}{4}$ liter, so subtraction is required.
3. $\frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$ liter.

Verified result: **INCORRECT**

CHOOSE A METHOD

UNIT 13

A quantity is two thirds of three fifths of a kilogram. Choose: A) add; B) multiply; C) divide. Select from the relationship, then verify exactly.

1. Choose B.
2. The phrase describes taking $\frac{2}{3}$ of $\frac{3}{5}$, so $\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$.

Verified result: **B**

INDEPENDENT PRACTICE

UNIT 13

A $\frac{3}{4}$ -meter ribbon is cut into pieces that are each $\frac{1}{8}$ meter long. Determine the number of pieces. Choose and execute the operation without a hint.

1. The question asks how many $\frac{1}{8}$ -meter pieces fit into $\frac{3}{4}$ meter.
2. That is division: $\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \times 8 = 6$.
3. Six pieces can be cut.

Verified result: **6**

APPLY & CONNECT

UNIT 13

A route is $\frac{7}{8}$ km long and $\frac{1}{3}$ km is removed from the distance still to cover. Use the number-line model to formulate the relationship, solve it, and interpret the remaining distance.

1. The planned route is $\frac{7}{8}$ km and $\frac{1}{3}$ km is removed from the remaining distance.
2. $\frac{7}{8} - \frac{1}{3} = \frac{21}{24} - \frac{8}{24} = \frac{13}{24}$ km.
3. The number-line marks are consistent with the exact remaining distance.

Verified result: **$\frac{13}{24}$**



QUICK RECALL

UNIT 13

Without looking back, simplify $20/30$ to lowest terms.

1. $20/30$ simplifies by 10 to $2/3$.

Verified result: $2/3$

QUICK RECALL

UNIT 13

Without looking back, subtract $1/3$ from $5/6$ and simplify.

1. $5/6 - 1/3 = 5/6 - 2/6 = 3/6 = 1/2$.

Verified result: $1/2$

QUICK RECALL

UNIT 13

Without looking back, compute $3/4 \div 1/8$.

1. $3/4 \div 1/8 = 3/4 \times 8/1 = 6$.

Verified result: 6

SPOT THE ERROR

UNIT 14

Diagnostic: a student sees the word 'after' and automatically multiplies when $1/4$ is poured out of $3/4$. Identify the reasoning error and give the exact result.

1. The claim is incorrect: a keyword is not an operation rule.
2. The relationship removes $1/4$ from $3/4$, so $3/4 - 1/4 = 1/2$.

Verified result: INCORRECT



CHOOSE A METHOD

UNIT 14

Cumulative strategy check: two thirds of three fourths is described. Choose A) add, B) multiply, or C) divide, then justify from the relationship.

1. Choose B because 'two thirds of three fourths' is a scaling relationship.
2. $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$.

Verified result: B

INDEPENDENT PRACTICE

UNIT 14

Complete this four-part cumulative set without hints: compare $\frac{5}{8}$ and $\frac{2}{3}$; add $\frac{1}{3} + \frac{1}{4}$; divide $\frac{3}{4}$ by $\frac{1}{8}$; convert $\frac{11}{4}$ to a mixed number.

1. Compare: $\frac{5}{8} < \frac{2}{3}$.
2. Add: $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$.
3. Divide: $\frac{3}{4} \div \frac{1}{8} = 6$.
4. Mixed conversion: $\frac{11}{4} = 2 \frac{3}{4}$.

Verified result: verified by deterministic truth record

APPLY & CONNECT

UNIT 14

Use the number line only as a model: convert $\frac{7}{4}$ to a mixed number and explain why both notations identify the same point.

1. $\frac{7}{4}$ contains one whole $\frac{4}{4}$ plus $\frac{3}{4}$.
2. Therefore $\frac{7}{4} = 1 \frac{3}{4}$, and the number-line point verifies the same value.

Verified result: $1 \frac{3}{4}$

QUICK RECALL

UNIT 14

Retrieve the original meaning of $\frac{3}{5}$: state what it means relative to one whole.

1. $\frac{3}{5}$ means three equal fifth-parts of the stated whole, and also the quotient $3 \div 5$.

Verified result: $\frac{3}{5}$



QUICK RECALL

UNIT 14

Represent $\frac{2}{5}$ as equal parts of one whole without looking back.

1. $\frac{2}{5}$ is represented by two of five equal parts of one whole.

Verified result: $\frac{2}{5}$

QUICK RECALL

UNIT 14

Decide whether $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent and justify exactly.

1. $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent because they are the same exact rational value.

Verified result: $\frac{4}{6}$

QUICK RECALL

UNIT 14

Simplify $\frac{18}{24}$ to lowest terms.

1. $\frac{18}{24}$ simplifies by 6 to $\frac{3}{4}$.

Verified result: $\frac{3}{4}$

QUICK RECALL

UNIT 14

Add $\frac{1}{3} + \frac{1}{6}$ and simplify.

1. $\frac{1}{3} = \frac{2}{6}$, so $\frac{2}{6} + \frac{1}{6} = \frac{1}{2}$.

Verified result: $\frac{1}{2}$



QUICK RECALL

UNIT 14

Multiply $\frac{2}{3} \times \frac{3}{4}$ and simplify.

1. $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$.

Verified result: $\frac{1}{2}$

QUICK RECALL

UNIT 14

Convert $\frac{11}{4}$ to a mixed number.

1. $\frac{11}{4} = 2 \frac{3}{4}$.

Verified result: $2 \frac{3}{4}$